

A TWO CHANNEL CALCULATION OF SCREENING CORRECTIONS

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Abstract:

We present a two channel eikonal calculation in which the rescattering through a diffractive channel is included in addition to the elastic channel. Considering the spread of the experimental data, we find that we can obtain a very good description of σ_{tot} , σ_{el} and B_{el} in the ISR - Tevatron energy range. In this range of energy the diffractive channel, that was included in our calculation, leads to a ratio of σ_{SD}/σ_{el} which varies between 1 and 0.5 for $20\text{ GeV} \leq \sqrt{s} \leq 14\text{ TeV}$ in agreement with the experimental data. The calculated survival probability of dijet production with a large rapidity gap is consistent with the data.

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1 Introduction

A simple Regge-pole parameterization suggested by Donnachie and Landshoff (DL)[1], has achieved remarkable success in describing hadronic and photon induced total and elastic cross sections. The elastic amplitude is, obviously, bounded by s-channel unitarity requiring screening (shadowing) corrections (SC) at sufficiently high energies. However, up to the Tevatron energies no decisive experimental signature of this effect has, thus far, been seen in the elastic channel.

An overall view of the features of high energy collisions [2][3][4][5][6][7] reveals at least two channels which are not compatible with the DL parameterization.

1. Since the total cross section behaves at high energies as s^{Δ_P} , where $\alpha_P(0) = 1 + \Delta_P$, we expect both σ_{el} and σ_{SD} , the integrated single diffraction (SD) cross section, to behave approximately like $s^{2\Delta_P}$ (since the Pomeron trajectory slope is small we can neglect the effects of the t integration). This is, indeed, the experimentally observed behaviour of σ_{el} , but σ_{SD} has a much milder energy dependence clearly seen in the ISR-Tevatron energy range[3].
2. $\langle |S|^2 \rangle$, which is defined[8] as the survival probability of two jets with a large rapidity gap (LRG), is measured[4][5] to be a small fraction of unity decreasing with energy. In a DL type model, containing very weak rescattering corrections, we expect $\langle |S|^2 \rangle$ to be approximately one with no, or very little, dependence on energy.

These problems have been discussed in our previous publications[9][10][11][12][13] where we have utilized a simplified one channel eikonal approximation in which the b-space scattering amplitude is given by

$$a(s, b) = i \left(1 - e^{-\frac{\Omega(s, b)}{2}} \right) , \quad (1)$$

the opacity is assumed to have a Gaussian form

$$\Omega(s, b) = \nu(s) e^{-\frac{b^2}{R^2(s)}} . \quad (2)$$

Even though this may be considered as an over simplified toy model, it provides a semi realistic reproduction of the main experimental features. Our main conclusion[11][12] is that while SC saturate the elastic channel in the TeV range, diffractive channels are saturated at relatively low (ISR) energies. This feature results in a mild energy dependence of σ_{SD} at

fixed M^2 , where the power behaviour of the cross section with s approaches a lns dependence in the high energy limit. The above eikonal approximation also leads[10][13] to estimates of $\langle |S|^2 \rangle$ which are compatible with the Tevatron[4] and HERA[6] data.

The one channel eikonal approach contains two severe deficiencies::

1. The main input assumption in such a model is that the rescattering is elastic, i.e. $\sigma_{diff} \ll \sigma_{el}$. This is clearly not corroborated by the data in the ISR-Tevatron energy range.
2. The introduction of SC introduces a fundamental problem in the definition of σ_{SD} . In a missing mass experiment, the integrated SD cross section is defined as

$$\sigma_{SD} = \int_{t_{min}(M^2)}^{t_{max}(M^2)} \int_{M_{min}^2}^{M_{max}^2} \frac{d^2 \sigma_{SD}}{dM^2 dt} , \quad (3)$$

where the upper M^2 integration limit is taken as a fixed fraction of s , usually $0.05s$. In a non screened triple Regge approximation [14] σ_{SD} behaves like $s^{2\alpha_P(t)-2}$ at fixed M^2 . With a super critical Pomeron ($\alpha_P > 1$), the M^2 integration converges, leading to an approximate $s^{2\Delta_P}$ behaviour of the integrated cross section. However, once SC are introduced[11], the M^2 dependence of σ_{SD} changes in the asymptotic limit to lns . Accordingly, the M^2 integration is divergent in s due to the upper M^2 integration limit. The predictions are, thus, flawed if examined over a wide energy range. Indeed, while we managed to fit σ_{SD} in the UA(4)-Tevatron range, our calculated σ_{SD} values [11] in the ISR energy range fall below the data [3].

2 The Model

Our goal is to construct an eikonal model in which the rescattering can be either elastic or diffractive. In the simplest approximation we consider the diffractively produced hadrons as a single hadronic state. We have, thus, two orthogonal wave functions

$$\langle \Psi_h | \Psi_D \rangle = 0 \quad (4)$$

where Ψ_h is the wave function of the incoming hadron, and Ψ_D is the wave function of the outgoing diffractively produced particles, initiated by this hadron.

Consider two wave functions Ψ_1 and Ψ_2 which are diagonal with respect to \mathbf{T} , the interaction operator. The amplitude of the high energy interaction is equal to

$$A_{i,k} = \langle \Psi_i \Psi_k | \mathbf{T} | \Psi_{i'} \Psi_{k'} \rangle = A_{i,k} \delta_{i,i'} \delta_{k,k'} . \quad (5)$$

In a two channel model $i, k = 1, 2$. The extension to a higher number of channels is straight forward. The amplitudes $A_{i,k}$ satisfy the diagonal unitarity condition

$$2 \operatorname{Im} A_{i,k}^{el}(s, b) = |A_{i,k}^{el}(s, b)|^2 + G_{i,k}^{in}(s, b) \quad (6)$$

for which we write the solution

$$A_{i,k}^{el}(s, b) = i \left(1 - e^{-\frac{\Omega_{i,k}(s,b)}{2}} \right) ; \quad (7)$$

$$G_{i,k}^{in}(s, b) = 1 - e^{-\Omega_{i,k}(s,b)} , \quad (8)$$

where $\Omega_{i,k}$ is the opacity of the (i, k) -th channel with a wave function $\Psi_i \times \Psi_k$.

In this representation Ψ_h and Ψ_D can be written as

$$\Psi_h = \alpha \Psi_1 + \beta \Psi_2 ; \quad (9)$$

$$\Psi_D = -\beta \Psi_1 + \alpha \Psi_2 \quad (10)$$

Since $|\Psi_h|^2 = 1$, we have

$$\alpha^2 + \beta^2 = 1 \quad (11)$$

The wave function of the final state is

$$\Psi_f = |T| \Psi_h \times \Psi_h > = \quad (12)$$

$$\alpha^2 A_{1,1} \Psi_1 \times \Psi_1 + \alpha \beta A_{1,2} \{ \Psi_1 \times \Psi_2 + \Psi_2 \times \Psi_1 \} + \beta^2 A_{2,2} \Psi_2 \times \Psi_2$$

We now define

$$a_{el}(s, b) = < \Psi_h \times \Psi_h | \Psi_f > = \alpha^4 A_{1,1} + 2 \alpha^2 \beta^2 A_{1,2} + \beta^4 A_{2,2} ; \quad (13)$$

$$a_{SD}(s, b) = < \Psi_h \times \Psi_D | \Psi_f > = \alpha \beta \{ -\alpha^2 A_{1,1} + (\alpha^2 - \beta^2) A_{1,2} + \beta^2 A_{2,2} \} \quad (14)$$

$$a_{DD}(s, b) = < \Psi_D \times \Psi_D | \Psi_f > = \alpha^2 \beta^2 \{ A_{1,1} - 2 A_{1,2} + A_{2,2} \} . \quad (15)$$

Assuming, that the double diffractive production is small, we have

$$a_{el}(s, b) = A_{1,1} - 2 \beta^2 (A_{1,1} - A_{1,2}) ; \quad (16)$$

$$a_{SD}(s, b) = -\alpha \beta (A_{1,1} - A_{1,2}) . \quad (17)$$

a_{el} and a_{SD} can be written in terms of the opacities $\Omega_1 \equiv \Omega_{1,1}$ and $\Omega_2 \equiv \Omega_{1,2}$

$$a_{el}(s, b) = i \left(1 - e^{-\frac{\Omega_1(s,b)}{2}} \right) - 2 \beta^2 \left(e^{-\frac{\Omega_1(s,b)}{2}} - e^{-\frac{\Omega_2(s,b)}{2}} \right) ; \quad (18)$$

$$a_{SD}(s, b) = -\alpha \beta \left(e^{-\frac{\Omega_1(s,b)}{2}} - e^{-\frac{\Omega_2(s,b)}{2}} \right) . \quad (19)$$

Defining $\Delta\Omega = \Omega_2 - \Omega_1$, we get

$$a_{el}(s, b) = i \left(1 - e^{-\frac{\Omega_1(s, b)}{2}} \right) - 2\beta^2 e^{-\frac{\Omega_1(s, b)}{2}} \left(1 - e^{-\frac{\Delta\Omega(s, b)}{2}} \right) ; \quad (20)$$

$$a_{SD}(s, b) = -\alpha\beta e^{-\frac{\Omega_1(s, b)}{2}} \left(1 - e^{-\frac{\Delta\Omega(s, b)}{2}} \right) . \quad (21)$$

In the limit where $\beta \ll 1$ and $\Delta\Omega \ll 1$, we reproduce the single channel eikonal model

$$a_{el}(s, b) = i \left(1 - e^{-\frac{\Omega_1(s, b)}{2}} \right) ; \quad (22)$$

$$a_{SD}(s, b) = \beta \frac{\Delta\Omega(s, b)}{2} e^{-\frac{\Omega_1(s, b)}{2}} . \quad (23)$$

For a convenient semi realistic parameterization, we follow our earlier publications and adopt a Gaussian approximation for the opacities

$$\Omega_1^P(s, b) = \frac{\sigma_{01}^P}{\pi(R_1^P(s))^2} \left(\frac{s}{s_0} \right)^{\Delta_P} e^{-\frac{b^2}{(R_1^P(s))^2}} = \nu_1^{0P} \left(\frac{s}{s_0} \right)^{\Delta_P} e^{-\frac{b^2}{(R_1^P(s))^2}} ; \quad (24)$$

$$\Delta\Omega^P(s, b) = \frac{\sigma_{0D}^P}{\pi(R_D^P(s))^2} \left(\frac{s}{s_0} \right)^{\Delta_P} e^{-\frac{b^2}{(R_D^P(s))^2}} = \nu_D^{0P} \left(\frac{s}{s_0} \right)^{\Delta_P} e^{-\frac{b^2}{(R_D^P(s))^2}} . \quad (25)$$

For both radii we assume the form

$$(R_i^P)^2 = (R_{0i}^P)^2 + 4\alpha'_P \ln(s/s_0) , \quad (26)$$

where $i = 1, D$, and take $s_0 = 1 \text{ GeV}^2$.

Since our investigation covers the ISR energy range , we need to include, in addition to the Pomeron parameters defined above, a Regge contribution which is defined in a similar way

$$\Omega_1^R(s, b) = \frac{\sigma_{01}^R}{\pi(R_1^R(s))^2} \left(\frac{s}{s_0} \right)^{-\eta} e^{-\frac{b^2}{(R_1^R(s))^2}} = \nu_1^{0R} \left(\frac{s}{s_0} \right)^{-\eta} e^{-\frac{b^2}{(R_1^R(s))^2}} ; \quad (27)$$

$$\Delta\Omega^R(s, b) = \frac{\sigma_{0D}^R}{\pi(R_D^R(s))^2} \left(\frac{s}{s_0} \right)^{-\eta} e^{-\frac{b^2}{(R_D^R(s))^2}} = \nu_D^{0R} \left(\frac{s}{s_0} \right)^{-\eta} e^{-\frac{b^2}{(R_D^R(s))^2}} ; \quad (28)$$

$$(R_i^R)^2 = (R_{0i}^R)^2 + 4\alpha'_R \ln(s/s_0) ; \quad (29)$$

where $\eta = 1 - \alpha_R(0)$.

In the following we check the ability of this simple model to describe the high energy data on

$$\sigma_{tot}(s) = 2\pi \int_0^\infty db^2 a_{el}(s, b) ; \quad (30)$$

$$\sigma_{el}(s) = \pi \int_0^\infty db^2 |a_{el}(s, b)|^2 ; \quad (31)$$

$$B_{el}(s) = \frac{\int_0^\infty db^2 b^2 a_{el}(s, b)}{2 \int_0^\infty db^2 a_{el}(s, b)} ; \quad (32)$$

$$\sigma_{SD}(s) = 2\pi \int_0^\infty db^2 |a_{SD}(s, b)|^2 ; \quad (33)$$

where we use $\Omega_i(s, b) = \Omega_i^P(s, b) + \Omega_i^R(s, b)$.

We assume single diffraction to be the only non negligible diffractive channel. By averaging the pp and $p\bar{p}$ data, we eliminate the need to discuss odd parity Regge exchanges, which may be marginally important at the lower ISR energies. The elimination of the odd contribution is exact for σ_{tot} and approximate for the elastic channel.

Once the parameters of our model are specified we can calculate the survival probability [8] of dijets with a LRG. We consider this is an important consistency check and discuss it in Section 4.

3 Data base and fitting procedures

Our study aims at a better understanding of the role s-channel unitarity screening and its relevance to soft Pomeron physics. Accordingly, we have limited our investigation to high energy pp and $p\bar{p}$ data at the ISR and above, i.e. $s > 300 \text{ GeV}^2$. This choice is made so as to minimize the dependence of our analysis on the secondary Regge exchanges.

Our data base contains 61 entries:

1. 18 values for σ_{tot} [2]. In the ISR range we have 10 points averaged between pp and $p\bar{p}$. Above ISR there are 8 measured values of $\sigma_{tot}(p\bar{p})$. These energies are sufficiently high to neglect the odd parity contributions.
2. 9 σ_{el} data points[2], 5 of which are averaged ISR values and 4 higher energy cross sections.
3. 11 values for B_{el} [2]. For 2 ISR energies, where we have the data, we took the averaged value.

4. 21 σ_{SD} data points[3], all of which were obtained from missing mass experiments initiated by $p\bar{p}$. The data corresponds to the sum of SD produced at either the proton or anti proton vertex. We have omitted from the analysis the SD data reported by Ref.[15]. These data points are systematically much lower than the other ISR reported SD cross sections[3].

Our model, as specified in Section 2, contains Pomeron and effective Regge trajectories

$$\alpha_P(t) = 1 + \Delta_P + 0.2t ; \quad (34)$$

$$\alpha_R(t) = 0.6 + t ; \quad (35)$$

where Δ_P is a fitted parameter. The assumed values of α_P , $\alpha_R(0)$ and α_R are those commonly used in Regge phenomenology. The other fitted parameters, defined in Section 2, are β , Δ_P , σ_{01}^P , $(R_{01}^P)^2$, σ_{0D}^P , σ_{01}^R , $(R_{01}^R)^2$ and σ_{0D}^R . In accordance with the triple Regge formalism we take

$$(R_{0D}^i)^2 = \frac{1}{2} (R_{01}^i)^2 \quad \text{where } i = P, R , \quad (36)$$

We neglect the triple vertex radius[11].

The predictions of our best fit, compared with the data are presented in Figs 1-4. The values of the fitted parameters are given in Table I. Our best fit has an overall $\chi^2 = 129.8$ corresponding to $\frac{\chi^2}{d.f.} = 2.6$. This is a non satisfactory high value. However, when inspecting the χ^2 contribution of the individual points we find that 7 points produce $\chi^2 = 65.2$. Neglecting these 7 points results in $\frac{\chi^2}{d.f.} = 1.5$. Re-fitting the data without these 7 points, we get essentially the same fit. Considering the spread of the experimental points reported with small quoted errors, we consider our fit to be a good one.

4 Survival probabilities

A LRG process is identified by the absence of produced hadrons in a sufficiently large rapidity gap region. This is regarded as a reliable signature for a crossed channel exchange of a Pomeron or alternatively a colourless gluonic state, be it non perturbative or perturbative. Recent experimental investigations of jets produced in the Tevatron[4][5] and HERA[6][7] have recorded this phenomena. We shall discuss here the simplest process where we have two jets produced with a LRG separation.

The experimentally measured quantity of interest is f_{gap} , which is the ratio of the cross section of dijet production with a LRG, and the inclusive two jet cross section (see Fig. 5)

$$f_{gap} = \frac{\sigma_{LRG}(2 \text{ jets})}{\sigma_{incl}(2 \text{ jets})} = \langle |S|^2 \rangle \cdot F_s , \quad (37)$$

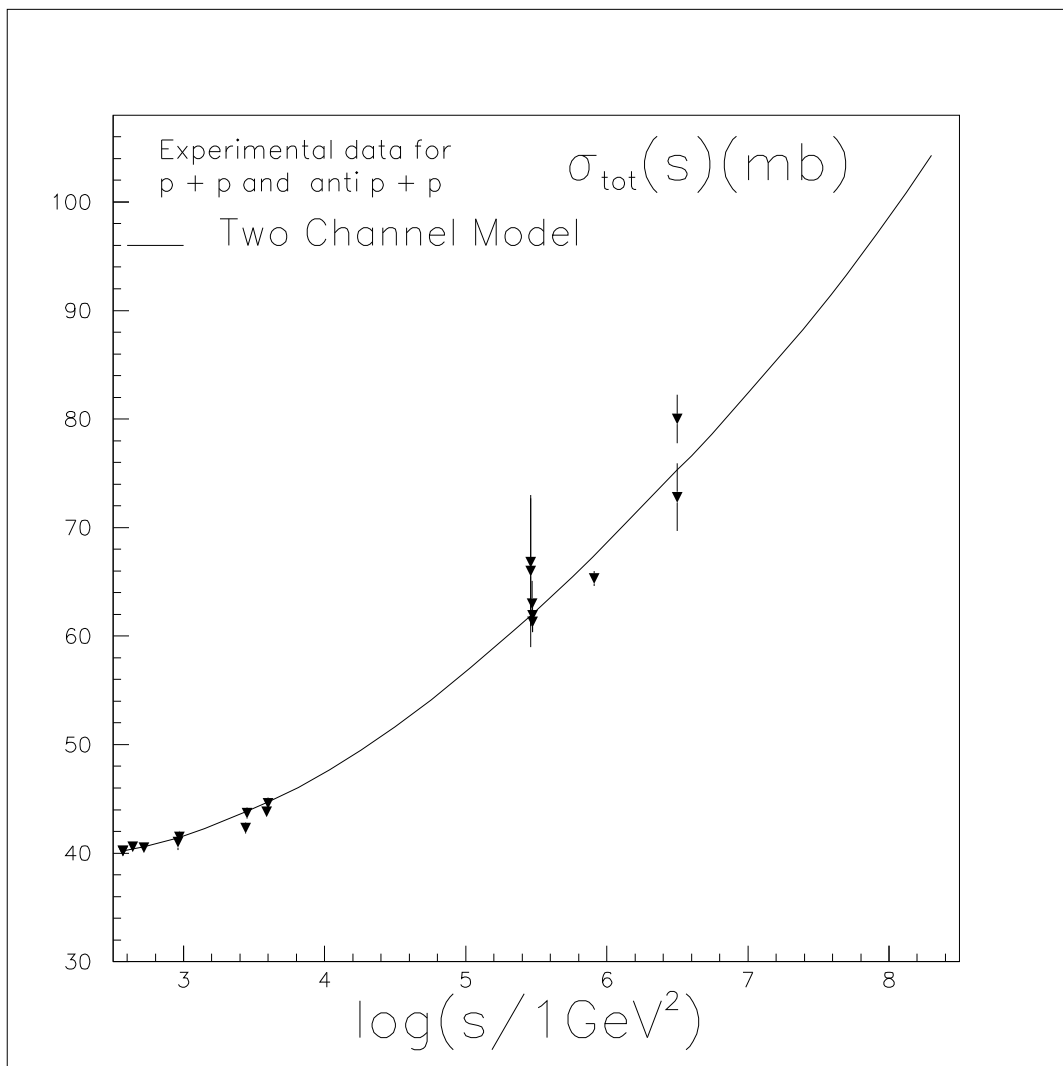


Figure 1: *Total pp and $\bar{p}p$ cross sections versus energy.*

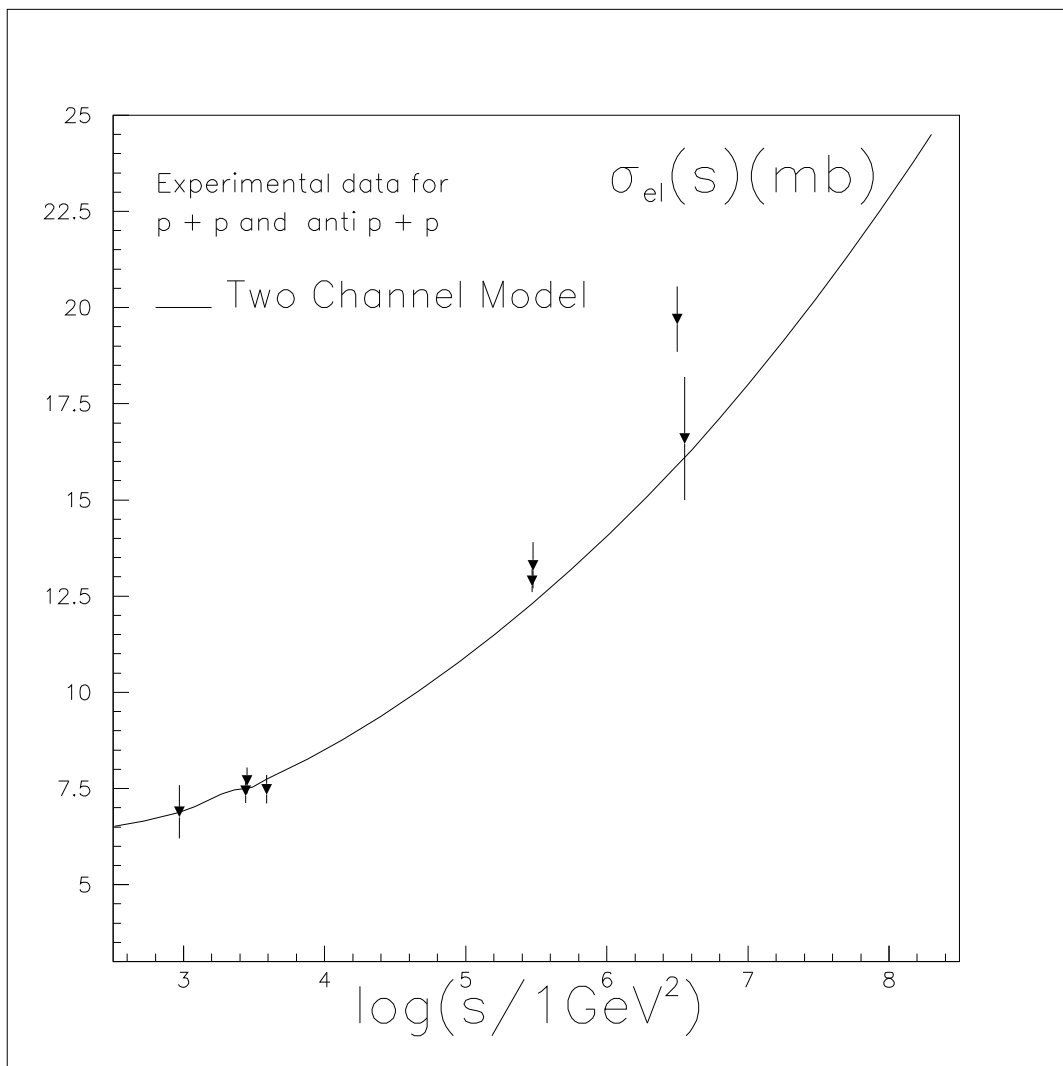


Figure 2: *Elastic cross section versus energy.*

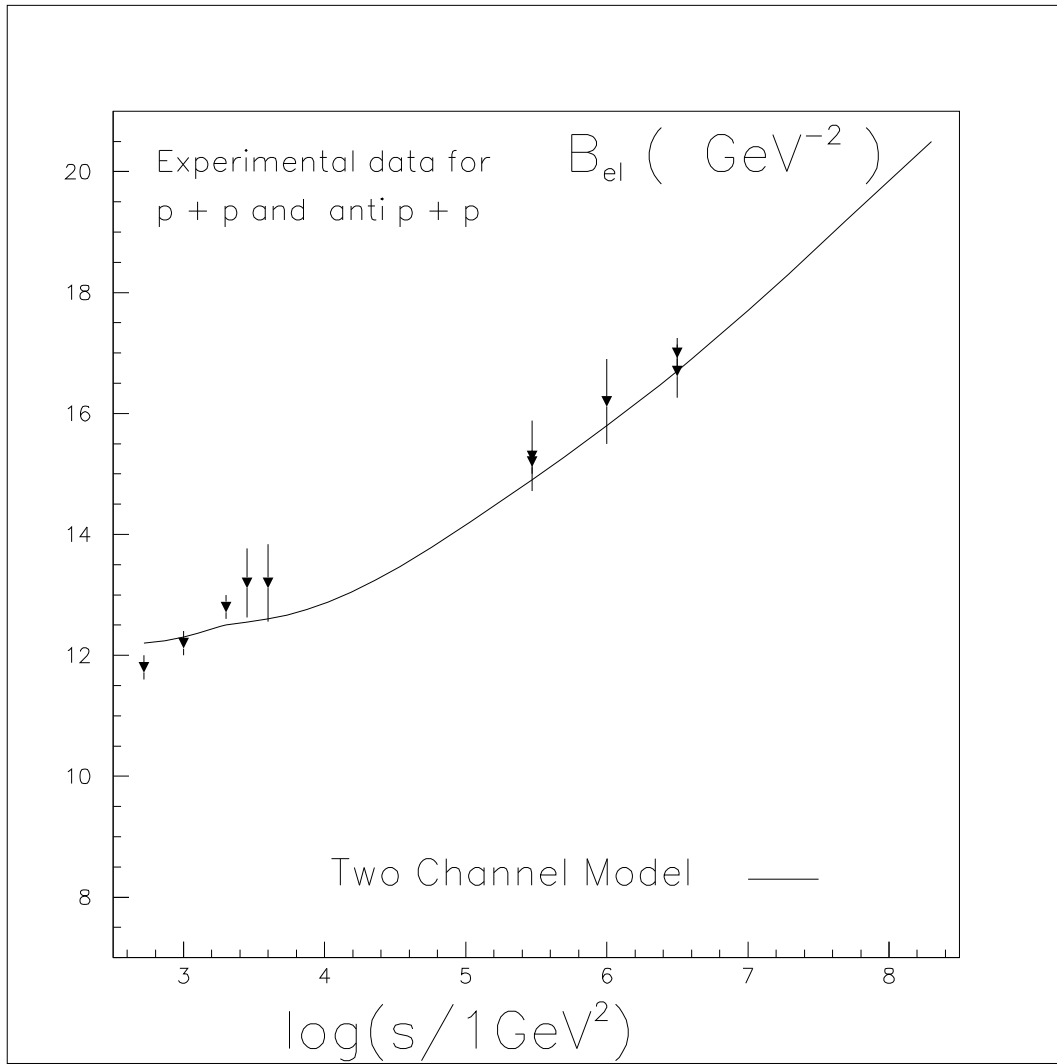


Figure 3: *Slope of the elastic cross section versus energy.*

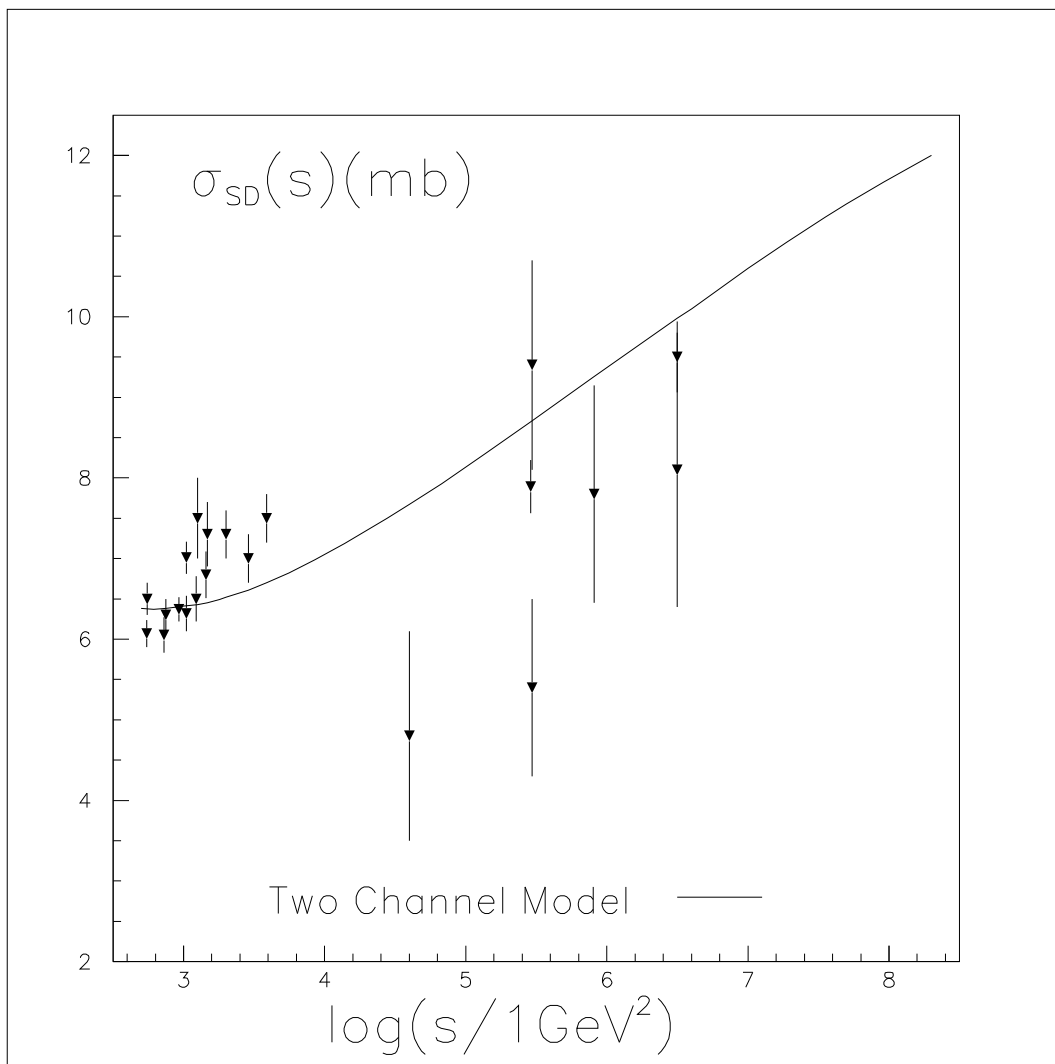


Figure 4: *Single diffraction cross section versus energy.*

Parameters	Best fit values
Δ_P	0.126
β	0.464
$\sigma_{01}^P (GeV^{-2})$	12.99
$(R_{01}^P)^2 (GeV^{-2})$	16.34
$\sigma_{0D}^P (GeV^{-2})$	145.6
$\sigma_{01}^R (GeV^{-2})$	4.78
$(R_{01}^R)^2 (GeV^{-2})$	12.44
$\sigma_{0D}^R (GeV^{-2})$	999.0

Table 1: *The values of the parameters of the Pomeron and Reggeon input Eikonal amplitudes.*

$$\mathbf{f}_{\text{gap}} = \frac{\sigma(\text{LRG})}{\sigma(\text{INCL})} = \langle S^2 \rangle \frac{\left| \begin{array}{c} \text{Diagram with } P \text{ (singlet exchange)} \end{array} \right|^2}{\left| \begin{array}{c} \text{Diagram with } G \text{ (octet exchange)} \end{array} \right|^2}$$

Figure 5: *Pictorial definition of f_{gap} , where P and G represent, respectively, the exchange of a colour singlet and a colour octet.*

F_s is the dynamical ratio of singlet to octet colour exchange leading to two jet production and its calculation is beyond the scope of this paper. F_s has to be modified by the survival probability $\langle |S|^2 \rangle$, which is the probability that the produced LRG event is not filled by hadronic debris resulting from rescattering of partons or hadrons.

The calculation of $\langle |S|^2 \rangle$ is model dependent. We shall follow the procedure suggested by Bjorken[8], in which the survival probability is the normalized b-space convolution of the hard partonic scattering amplitude and $P(s, b)$ - the probability that the two initial projectiles do not interact inelastically.

$$\langle |S(s)|^2 \rangle = \frac{\int d^2b P(s, b) a_H(\Delta y, b)}{\int d^2b a_H(\Delta y, b)}. \quad (38)$$

In the following we neglect the dependence of a_H on the rapidity gap of interest.

In a single channel eikonal model

$$P(s, b) = e^{-\Omega(s, b)}. \quad (39)$$

Taking a Gaussian approximation for both $\Omega(s, b)$ and $a_H(s, b)$ simplifies the calculation and we obtain

$$\langle |S(s)|^2 \rangle = \frac{a\gamma[a, \nu]}{\nu^a}, \quad (40)$$

where $\gamma(a, x) = \int_0^x z^{a-1} e^{-z} dz$, ν is defined in Eq. (2) and $a = \frac{R_s^2(s)}{R_H^2(s)}$.

The results of our single channel eikonal calculation[10] are in fair agreement with the D0 data[4]. However, the calculated energy dependence is not sufficiently strong. This deficiency is removed once we estimate[13] ν , R_s^2 and R_H^2 directly from the data. The two channel eikonal model presented in this paper may have some problems in reproducing the experimental energy dependence, since the data for $\frac{\sigma_{el} + \sigma_{diff}}{\sigma_{tot}}$ is almost energy independent in the ISR-Tevatron energy range, whereas $\frac{\sigma_{el}}{\sigma_{tot}}$ grows monotonically with s . For this reason we consider the calculation of $\langle |S|^2 \rangle$ in a multi channel eikonal model, to be an important check of the validity of this approach to calculating the SC.

A hard LRG b-space amplitude, uncorrected by the survival probability, is given in our formalism by

$$a_H = \alpha^2 a_1^H + \beta^2 a_2^H \quad (41)$$

where a_1^H and a_2^H correspond to processes initiated by our Ψ_1 and Ψ_2 wave functions. As previously, we impose the restriction that no inelastic rescattering modifies our hard process, by multiplying each component of a_H by the corresponding $e^{-\Omega_n}$.

The survival probability in our two channel model is given by:

$$\langle |S(s)|^2 \rangle = \frac{N(s)}{D(s)}, \quad (42)$$

where

$$D(s) = \pi \int_0^\infty db^2 \left(\sum_{i=1}^2 a_i^H P_i(s, b) \right) = \quad (43)$$

$$\pi \int_0^\infty db^2 \left(e^{-\Omega_1(s, b)} \{ 1 - \beta^2 [1 - e^{-\Delta\Omega(s, b)}] \} a_1^H + \beta^2 e^{-\Omega_1(s, b)} e^{-\Delta\Omega(s, b)} a_{SD}^H \right);$$

and

$$N(s) = \pi \int_0^\infty db^2 \left(\sum_{i=1}^2 a_i^H \right) = \pi \int_0^\infty db^2 \left(a_1^H + a_{SD}^H \right). \quad (44)$$

$\langle |S(s)|^2 \rangle$ can be calculated numerically once our free parameters have been fixed provided we know R_H^2 . We have checked that taking $R_H^2 = 8 \text{ GeV}^{-2}$ [16], we obtain $\langle |S(s)|^2 \rangle$ values in the Tevatron range which are in a fair agreement with the D0 data [4]. The calculated energy dependence of $\langle |S(s)|^2 \rangle$ is given by

$$\frac{\langle |S(\sqrt{s} = 630 \text{ GeV})|^2 \rangle}{\langle |S(\sqrt{s} = 1800 \text{ GeV})|^2 \rangle} = 1.3 - 1.4$$

to be compared with D0 value of 2.2 ± 0.8 . In as much as this is just a consistency check and not an actual fit, we consider these results to be reasonable. A improved and detailed analysis of LRG events in a multi channel eikonal model will be published soon [17].

5 Discussion

The following are the main features and predictions of our suggested model.

1. The consistent treatment of the diffractive dissociation channel in our model leads to a good description of the experimental data on the single diffraction in ISR - Tevatron energy range. We reproduce the energy dependence of the ratio σ_{SD}/σ_{el} as well as its value (see Fig. 8a) in agreement with the experimental data.
2. Our input Pomeron has $\Delta_P = 0.126$ which determines our high energy predictions (see Figs. 1 - 4). For LHC, $\sqrt{s} = 14 TeV$, we predict $\sigma_{tot} = 104.3 mb$. Our prediction is slightly higher than the DL prediction of $101.5 mb$ and is compatible with a recent prediction of Block et al.[18] of $108 \pm 3.4 mb$. The model suggested in Ref. [18] is a single channel eikonal calculation with a Regge input. DL approach is a non screened Regge model corrected by a weak Pomeron - Pomeron cut. The small differences between the σ_{tot} predictions at LHC energies of these rather different models, re-enforces our observation that SC are weak for σ_{tot} .
3. Our LHC predictions for the elastic channel are $\sigma_{el} = 24.5 mb$ and $B_{el} = 20.5 GeV^{-2}$. Our σ_{el} prediction is about 15% lower than the prediction of Ref. [18]. This is compatible with the small difference between the two σ_{tot} predictions, combined with our predicted value of B_{el} being higher than the prediction of Block et al.
4. Some insight into the difference between a non screened DL type b -space elastic amplitude, and our screened $a_{el}(s, b)$ is provided Fig.6. As seen in Fig.6a the DL non screened amplitude violates s -channel unitarity (to be distinguished from the Froissart bound) at small impact parameters starting from $\sqrt{s} \simeq 3 TeV$. The introduction of a weak P-P cut [1] reduces $a_{el}^{DL}(s, b)$ at small b by approximately 10%, leaving the DL prediction for σ_{tot} practically unchanged. In comparison, our screened $a_{el}(s, b)$ is considerably lower and wider (see Fig.6b). This behaviour is a consequence of both screening and the existence of a competing diffractive channel.
5. As we have noted, the investigation of σ_{SD} in the ISR-Tevatron energy range provides strong support for the importance of SC at existing collider energies. It is interesting to compare our approach with the Pomeron flux renormalization suggested by Goulianos [19]. In the eikonal model, the damping due to unitarity in the diffractive channel is given by $e^{-\Omega_i(s, b)}$, where i denotes the rescattering channel considered. As such, the damping is b -dependent, with maximum suppression at small b resulting from the fact

that the input $\Omega_i(s, b)$ are central. As we saw, the damping is smooth and becomes significant at ISR energies. In Ref.[19], a unitarity flux correction is applied only when the Pomeron flux exceeds unity. This amelioration is b -independent. Numerically, our σ_{SD} has a stronger energy dependence than suggested in Ref. [19]. In the ISR-Tevatron range the difference between the predictions is small. However, at LHC, $\sqrt{s} = 14 TeV$, we predict $\sigma_{SD} = 12 mb$ to be compared with Goulianos' estimate of $10.5 mb$.

6. A simple Gaussian input for the opacity leads to dips in $\frac{d\sigma_{el}}{dt}$ at lower values of t than in the data. To obtain these dips at their observed experimental values, the Gaussian form of $\Omega(s, b)$ has to be replaced by a dipole form[20]. We consider this to be a relatively minor technical modification of the calculation. We note that the dip structure can be reproduced in an eikonal calculation[20], as well as by the introduction of a weak P-P cut[1], leaving the DL results on σ_{tot} and σ_{el} virtually unchanged.
7. The distinctive features of our diffractive b -space amplitude are visible in Fig.7. Our $a_{SD}(s, b)$ is peripheral. This is to be compared with Goulianos et al. [19] who produce a SD amplitude proportional to the unscreened DL amplitude shown in Fig.6a. Our model is, thus, compatible with the Pumplin's bound [21]

$$|a_D(s, b)|^2 = |a_{el}(s, b)|^2 + |a_{SD}(s, b)|^2 \leq 1 , \quad (45)$$

which translates, after a b -integration, to

$$\sigma_{el}(s) + \sigma_{SD}(s) \leq \frac{1}{2} \sigma_{tot}(s) . \quad (46)$$

In Fig.8 we show the energy dependence of $R_{el} = \frac{\sigma_{el}}{\sigma_{tot}}$, $R_{SD} = \frac{\sigma_{SD}}{\sigma_{tot}}$ and $R_D = R_{el} + R_{SD}$ (Fig.8a), and the energy dependence $|a_D(s, b)|^2 = |a_{el}(s, b)|^2 + |a_{SD}(s, b)|^2$ at $b=0$ (Fig.8b). All ratios are well below the Pumplin bound. We have checked that the bound is saturated at the non realistic ultra high energies of $\sqrt{s} \simeq 10^9 GeV$. The Goulianos model differs in as much as it violates the Pumplin bound at small b . This is a consequence of both $a_{el}(s, b)$ and $a_{SD}(s, b)$ peaking at $b = 0$. We note that the Pumplin's bound was proven in a multichannel eikonal model. Its validity in other screening models is not clear [22]. A peripheral b -space structure for a_{SD} implies dips or breaks in $\frac{d\sigma_{SD}}{dt}$ at small t , where the details depend on form of the input taken in the eikonal calculation. Some indications for such a structure have been observed in ISR SD experiment with a very small diffractive mass [23]. More accurate experimental data are required to investigate this aspect further.

8. A more detailed treatment of $\langle |S|^2 \rangle$ in a multichannel eikonal model is in preparation [17]. In the present paper we have calculated this observable as a check of our model.

We obtain a reasonable energy dependence of $\langle |S|^2 \rangle$ even though R_D is almost energy independent in the ISR- Tevatron energy range (see Fig.8a). The resulting energy dependence of $\langle |S|^2 \rangle$ in our model is not surprising as each channel is suppressed by $e^{-\Omega_i(s,b)}$. As long as the SC are small, the energy dependence of $\langle |S|^2 \rangle$ is determined by R_D which is almost flat in the energy range of interest. When the SC become significant, the relevant ratios are R_{el} and R_{SD} , both of which show sufficient energy dependence to induce a final $\langle |S|^2 \rangle$ which is compatible with the D0 measurements [4].

9. In a recent preprint Cudell et al.[24] come to a conclusion similar to ours for σ_{tot} . However, Cudell et al. use this result to question whether unitarity corrections are necessary at all. We wish to emphasize again that the value of SC differs for different channels. They are mild for the elastic channels but appreciable for σ_{SD} and $\langle |S|^2 \rangle$. Since we believe that the same colour singlet (Pomeron) mediates all these reactions, we advocate not drawing general conclusions which are based on the finding in one channel only.

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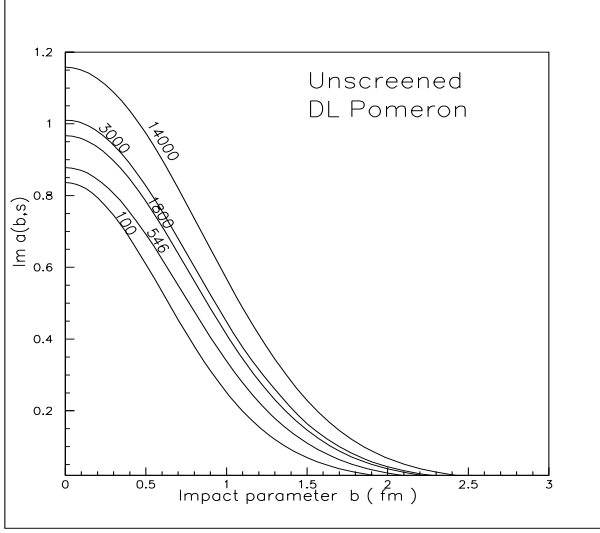


Fig. 6-a

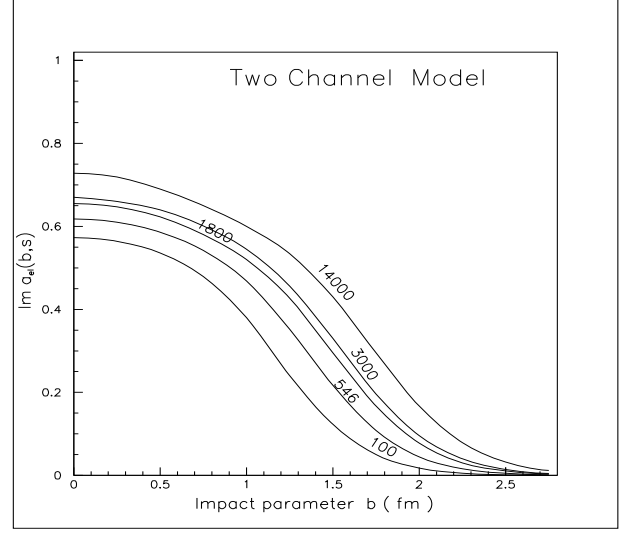


Fig. 6-b

Figure 6: The b - dependence of the elastic amplitude for the unscreened DL Pomeron (Fig.6a) and in our two channel model (Fig. 6b).

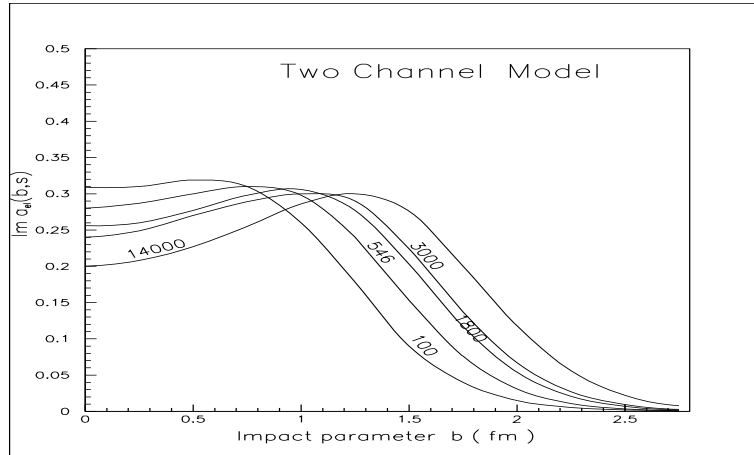


Figure 7: The b - dependence of the single diffraction amplitude in two channel model.

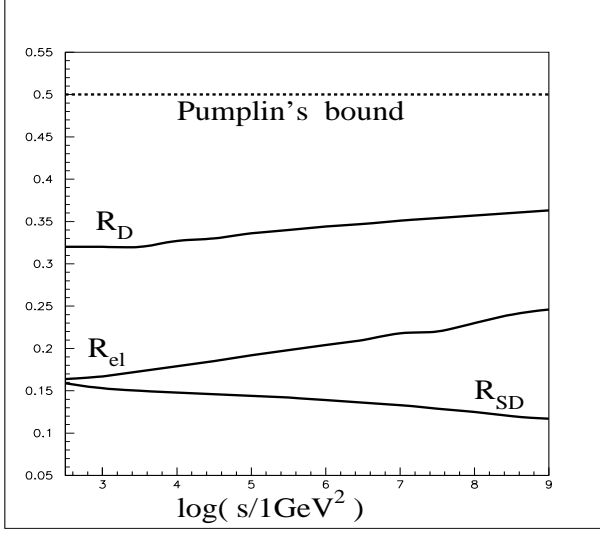


Fig. 8-a

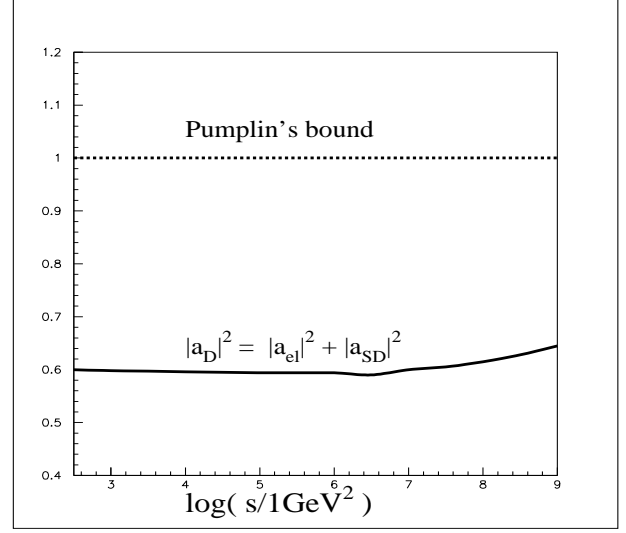


Fig. 8-b

Figure 8: *Energy behaviour of the ratios $R_D = (\sigma_{el} + \sigma_{SD})/\sigma_{tot}$, $R_{el} = \sigma_{el}/\sigma_{tot}$ and $R_{SD} = \sigma_{SD}/\sigma_{tot}$ (Fig.8a) and $|a_D(s, b = 0)|^2 = |a_{el}(s, b = 0)|^2 + |a_{SD}(s, b = 0)|^2$ (Fig.8b). The dotted line in both figures shows the Pumplin bound.*

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